Direct Analogy Between Mass And Heat Transfer to Beds of Spheres

ASHIS SEN GUPTA and GEORGE THODOS

Northwestern University, Evanston, Illinois

In the last twenty years considerable work has been reported in the literature on the simultaneous mass and heat transfer in the flow of fluids through packed and fluidized beds. In most of these studies the results have been presented in terms of j factors, which were introduced by Chilton and Colburn (*2*) and first used for packed beds in 1943 by Gamson, Thodos, and Hougen (*6*). These investigators studied the constant rate of drying of water from the surfaces of inert spherical and cylindrical particles of celite catalyst carriers into a stream of air flowing through the interstices of a packed bed of the particles. In their study different relationships between the mass transfer factor j_d and Reynolds number and between the heat transfer factor j_h and Reynolds number were obtained with a ratio $j_h/j_d = 1.076$. This ratio was found by Sherwood (*12*) to be consistent with the assumption that the surface temperature of the particles is the same as the wet-bulb temperature of the air flowing through the bed.

Since this initial study considerable information has been reported in the literature for both mass and heat transfer in which not only water but also several organic liquids have been evaporated from the surface of these particles (*7*). Sen Gupta and Thodos (*11*) have made a comprehensive review of the mass and heat transfer data available in the literature for the flow of gases through packed and distended beds of spheres and found that this information produced separate relationships between ϵj_d and between ϵj_h and Reynolds number with a ratio of $j_h/j_d = 1.076$. This discrepancy between the mass and heat transfer factors has caused considerable speculation, since theoretical considerations indicate that a direct analogy should exist for these two transfer processes. In the present study a critical review of the experimental conditions and assumptions have been conducted in an attempt to determine the possible sources of these differences.

Gamson, Thodos, and Hougen (*6*) assumed that the temperature prevailing on the surface of the particles was constant throughout the bed and equal to the wet-bulb temperature of the air. However De Acetis and Thodos (*4*) measured surface temperatures directly and concluded that this assumption is only approximately true for high mass flow rates. Although in the studies of Gamson et al. and of subsequent investigators (*13, 14*) considerable effort was exerted to obtain approximately adiabatic conditions, no attempt was made to account for the difference between the temperature of the wetted particles and that of the surroundings. Radiation calculations based on these differences indicate that approximately 2.5% of the total heat transfer is due to radiation. Therefore the quantity of heat transferred to the particles which was used in the calculation of the j_h factor represents the sum of convective heat transfer between the air stream and the particles and the radiation between the surrounding walls and the particles. Therefore the values of j_h calculated from these data should be somewhat greater than those which would have resulted if radiation effects had been eliminated.

Furthermore for the calculation of the mass transfer coefficient k_g, the log-mean partial pressure difference across the film was assumed to apply in all these studies. However in most cases this quantity represents an oversimplification of the actual pressure difference of the transferable component across the film. Therefore in the present study an attempt has been made to eliminate these sources that give rise to the discrepancies between j_d and j_h by measuring surface temperatures directly, by keeping the temperature of the surrounding walls at the same temperature as that of the wetted particles, and by applying the proper driving force for use in the calculation of k_g.

Bird, Stewart, and Lightfoot (*1*) present the following six conditions which must be satisfied for a direct analogy between mass and heat transfer: constant physical properties, relatively small rate of mass transfer, no chemical reaction in the fluid, no viscous dissipation, no emission or absorption of radiant energy, and no pressure diffusion, thermal diffusion, or forced diffusion. In the previous experimental work (*3, 4, 6, 7, 8, 13, 14*) all of these conditions have been satisfied with the exception of the second and fifth points. For an exact analogy, it is essential that the mass transport, and therefore the driving force, be as small as practically possible.

THEORETICAL BACKGROUND

In a two-component system composed of substances A and B the molar fluxes of these components are defined by Fick's first law as follows:

$$N_A - x_{A_o}(N_A + N_B) = c\mathcal{D}_{AB}\nabla x_A \qquad (1)$$

and

$$N_B - x_{B_o}(N_A + N_B) = c\mathcal{D}_{AB}\nabla x_B \qquad (2)$$

For the transfer of component A across a stagnant film of component B, $N_B = 0$. Therefore both Equations (1) and (2) reduce to

$$N_A = \frac{c\mathcal{D}_{AB}}{1 - x_{A_o}}\nabla x_A \qquad (3)$$

The rate of mass transfer of component A represented by Equation (1) can also be expressed in terms of a mass transfer coefficient k_x as follows:

$$N_A - x_{A_o}(N_A + N_B) = k_x \Delta x_A \qquad (4)$$

For the case $N_B = 0$ Equation (4) becomes

$$N_A = \frac{k_x}{1 - x_{A_o}}\Delta x_A \qquad (5)$$

For diffusion through a gas film Equation (5) is equivalent to

$$N_A = \frac{k_x}{p_B}\Delta p_A \qquad (6)$$

In numerous mass transfer studies (*3, 6, 13*), the following rate equation was used:

$$N_A = k_g \Delta p_A \tag{7}$$

From Equations (6) and (7) it follows that

$$k_x = k_g p_B \tag{8}$$

In 1935 Chilton and Colburn (*2*) introduced the j factor for mass transfer defined as

$$j_d = \frac{k_g \cdot p_{gf}}{G/M} \left(\frac{\mu}{\rho \mathcal{D}_{AB}} \right)_f^{2/3} \tag{9}$$

For small driving forces the partial pressure of the nontransferable component p_B is approximately equal to p_{gf}. Under these conditions Equation (9) becomes

$$j_d = \frac{k_x}{G/M} \left(\frac{\mu}{\rho \mathcal{D}_{AB}} \right)_f^{2/3} \tag{10}$$

For high rates of mass transfer the velocity profile in the gas film across which transfer is taking place becomes distorted, and therefore the mass transfer coefficient k_x is dependent on the rate of mass transfer. For relatively low mass transfer rates k_x is nearly independent of the rate, since the velocity profile does not undergo a significant distortion. In the development of Equation (10) the restriction of a small driving force across the gas film is required for k_x to be independent of the mass transfer rate.

The rate of heat transfer which occurs simultaneously with the mass transfer can be expressed as

$$q = h_g \Delta t \tag{11}$$

where Δt is the temperature potential across the gas film and h_g is the heat transfer coefficient which is included in the j factor for heat transfer:

$$j_h = \frac{h_g}{c_p G} \left(\frac{c_p \mu}{k} \right)_f^{2/3} \tag{12}$$

Chilton and Colburn (*2*) determined heat and mass transfer factors for the evaporation of water into air flowing countercurrent to it in a falling film tower. The resulting heat and mass transfer factors were found to be in substantial agreement and consistent with their theoretical deduction that

$$j_h = j_d \tag{13}$$

This analogy can be extended to include the flow of fluids through packed beds, since for this operation the concentration and temperature profiles are similar for relatively small mass transfer rates.

EXPERIMENTAL PROCEDURE AND EQUIPMENT

Mass transfer rates were determined by the evaporation of water from the surface of celite spheres into a stream of air passing through the voids of each bed. Each bed of spherical particles was saturated by soaking it with water for more than 2 hr. Three fixed beds of different void fractions were used and consisted of one packed bed ($\epsilon = 0.444$) and two distended beds ($\epsilon = 0.576$ and $\epsilon = 0.778$). Both distended beds consisted of approximately five layers of spheres, while the packed bed consisted of three layers of active spheres preceded by three additional layers of inert plastic spheres. The particles of each bed were frozen in space with short lengths of fine rigid wires and formed in all cases a body-centered cubic lattice. The details of the preparation of the beds are described elsewhere (*8*), including the precaution taken to eliminate wall effects by shaping the peripheral spheres to conform to the cylindrical geometry of the container.

The experimental apparatus consisted of a chamber which had a squared cross section (5 in. × 5 in.) and which was 7 in. in height. The entrance and exit sections of the chamber were gradually tapered to accommodate a 1-in. pipe through which air was blown downwardly through the bed. The chamber was provided with a shelf on which the cartridge containing the bed of spheres could be rested. Fine copper-constantan thermocouples (No. 36 gauge) were connected to twelve spheres located at the entrance, middle, and exit of each bed and permitted the direct measurement of the surface temperature of the spheres. The thermocouple wires were attached to a multipoint plug, mounted on the door of the chamber, which could be disengaged so that each bed could be removed for periodic weighings. The other end of the plug was connected to a potentiometer.

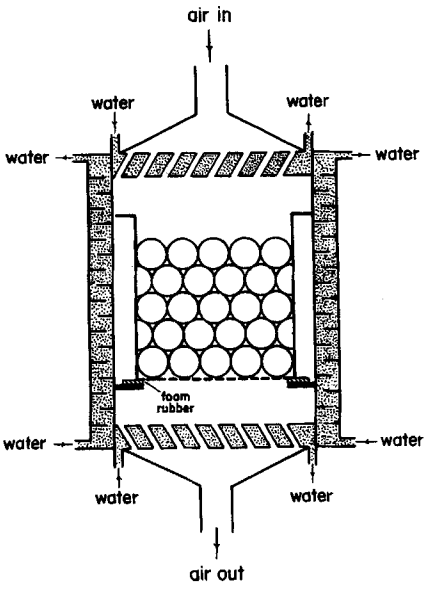

Fig. 1. Cross-sectional view of reactor chamber and water jacketing facilities.

In order to eliminate radiation effects the walls and the door of the chamber were water jacketed so their temperature could be maintained at the average surface temperature of the spheres by the adjustment of the flow of hot and cold water through the jacket. The horizontal walls at the top and bottom of the chamber were also water jacketed and were provided with a large number of oblique openings, which permitted the flow of air and prevented the upper and the lower surfaces of the bed from seeing any part of the equipment which was not maintained at the surface temperature of the spheres. The details of the chamber, bed, and water jacketing facilities are presented schematically in Figure 1.

In order to provide flexibility on the operating conditions a humidifier, a cooler, and an electric preheater were used to condition the air entering the chamber. The humidifier consisted of a 2-in. cylindrical glass tube, which was packed with solid plastic spheres. Water was allowed to flow concurrently with air from the top of the humidifier into a separation box located at the bottom where excess water was drained off.

A typical run consisted of subjecting a bed of spheres saturated with water to drying by forcing conditioned air through the interstices of each packing. At regular time intervals (which varied from 6 to 10 min.) the air flow was stopped and the bed was removed from the experimental chamber and was weighed. This procedure was repeated as long as a constant rate of drying prevailed. Altogether twenty-seven experimental runs were conducted; the results and basic information for each run are presented elsewhere (*9*).

RESULTS

The experimental results were used to calculate values of k_g and j_d for mass transfer and h_g and j_h for heat transfer. The procedure employed for the calculation of these quantities is identical to that described elsewhere (*6*), in which a log-mean partial pressure difference across

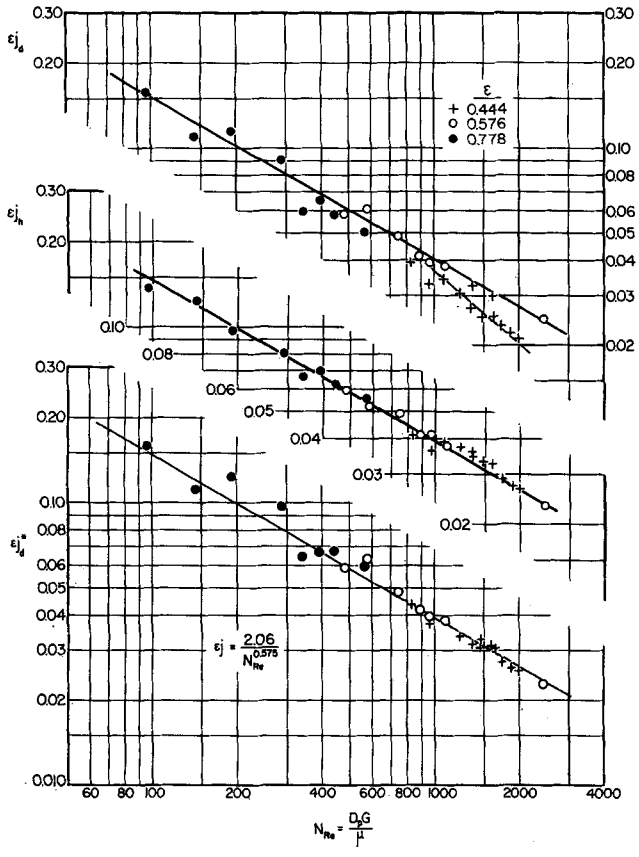

Fig. 2. Relationships between ϵj_d and N_{Re}, ϵj_h and N_{Re}, and ϵj^*_d and N_{Re} for packed and distended beds of spheres.

the gas film of the entire bed was used to calculate k_g. The Prandtl and Schmidt groups at average film conditions were found to be nearly constant at $N_{Pr} = 0.718$ and $N_{Sc} = 0.606$.

In order to include the effect of void volume a method outlined elsewhere (*11*) was adopted, and the products ϵj_d and ϵj_h were plotted vs. N_{Re} on log-log coordinates. The results of ϵj_d against N_{Re} are presented in Figure 2. It can be seen from this figure that considerable scatter is obtained, in particular with the values resulting from the packed bed which deviate linearly from the relationship representing the data of the two distended beds.

The correlation between ϵj_h and N_{Re} is also presented in Figure 2 and indicates that a good linear relationship exists for the variables investigated in this study. The excellent consistency of the heat transfer data and the poor correlation of the mass transfer data suggest that the mass transfer data may not have been properly analyzed. Except for the deviations of the packed bed data, the relationship of Figure 2 for ϵj_d vs. N_{Re} is identical to the ϵj_h vs. N_{Re} relationship.

This behavior indicates that the elimination of radiation effects coupled with the relatively low mass transfer rates of this study has produced essentially a one-to-one correspondence for mass and heat transfer.

Since these studies have been restricted to low rates of mass transfer, the partial pressure difference of the transferable component across the gas film was necessarily small. Consequently the resulting j_d values must be influenced by small errors in the magnitude of this potential. Despite the fact that in all similar studies mass transfer coefficients have been calculated by the use of a log-mean driving force, its use is not completely justified and particularly with low rates of mass transfer. Consequently the true-mean driving force prevailing across the gas film has been re-examined and has been applied to the analysis of the experimental data of this study. This expression in its complete form is

$$\Delta p^* = \frac{\frac{p_2}{\pi - p_2} - \frac{p_1}{\pi - p_1}}{\frac{\pi}{\pi - p_w}\left[\frac{1}{\pi - p_1} - \frac{1}{\pi - p_2} + \frac{1}{\pi - p_w}\ln\frac{(\pi - p_2)(p_w - p_1)}{(\pi - p_1)(p_w - p_2)}\right]} \tag{14}$$

and utilizes for its development the basic assumptions presented by Gamson (*5*). A more detailed and complete development for this expression will be found elewhere (*9*). On the assumption that $\pi - p_1 \approx \pi - p_2 \approx \pi - p_w$ and that $(\pi)/(\pi - p_w) \approx 1$

Equation (14) simplifies to the log mean partial pressure difference:

$$\Delta p_{ln} = \frac{(p_w - p_1) - (p_w - p_2)}{\ln\frac{p_w - p_1}{p_w - p_2}} \tag{15}$$

FINAL CORRELATION FOR HEAT AND MASS TRANSFER

The true-mean partial pressure difference expressed by Equation (14) has been used in this study to calculate mass transfer coefficients for the final correlation of the data. The resulting ϵj_d^* values were plotted against the Reynolds number to produce the straight-line relationship presented in Figure 2. The asterisk denotes j_d values resulting from the use of Equation (14). The ϵj_d^* vs. N_{Re} relationship of Figure 2 properly correlates the experimental data for both distended beds and the packed bed. Of particular interest is the fact that the packed bed data now properly extend the straight line relationship of ϵj_d^* vs. N_{Re} resulting from the two distended beds. The relationship of ϵj_d^* vs. N_{Re} for mass transfer is identical to the relationship ϵj_h vs. N_{Re} for heat transfer. In order to verify this analogy ϵj values for both transfer processes have been plotted against N_{Re} on log-log coordinates in Figure 3. It can be seen from this figure that values of ϵj for mass and heat transfer properly superimpose each other and produce the combined linear relationship on log-log coordinates. This relationship can be expressed analytically as follows:

$$\epsilon j = \frac{2.06}{N_{Re}^{0.575}} \tag{16}$$

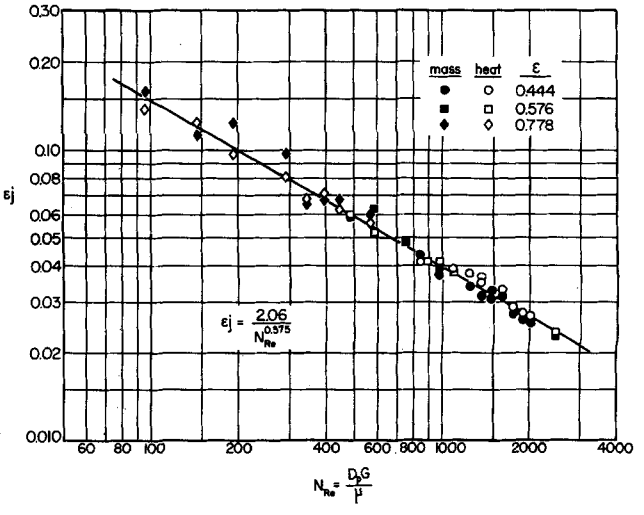

Fig. 3. Generalized relationship between ϵj and N_{Re} for both mass and heat transfer in the flow of fluids through packed and distended beds of spheres.

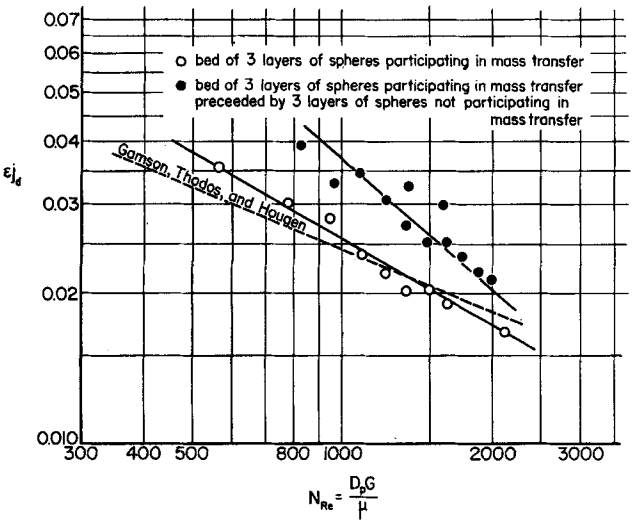

Fig. 4. Relationships between ϵj_d and N_{Re} for shallow and deep beds of spheres.

CONCLUSIONS

The results of this study verify experimentally for the first time that a direct correspondence between mass and heat transfer exists in the flow of fluids through packed and distended beds of spheres. Earlier attempts (*6, 13*) to approach this analogy have shown that the ratio of $j_h/j_d = 1.076$. This deviation from unity is due to experimental limitations resulting from relatively high rates of mass transfer, radiation effects, and the use of the log-mean partial pressure difference, which does not represent the actual mean driving force over the entire bed. This analogy can be extended to include packings of other geometrical configurations and aggregation state of the bed, as pointed out by Sen Gupta and Thodos (*10*).

A comparison of the ϵj values obtained in this study with those of Gamson, Thodos, and Hougen (*6*) shows that the ϵj_d values of this investigation are considerably higher, as indicated in Figure 4. In their studies Gamson, Thodos, and Hougen used relatively thin beds of spheres and consequently raised some doubts of the existence of typical bed conditions. This condition was essentially repeated in the present study by using a packed bed of only three layers of spheres. The data resulting from this thin bed are not much different from those reported by Gamson, Thodos, and Hougen as shown in Figure 4.

On the other hand the ϵj values that followed, with a bed consisting of three additional layers of inert plastic spheres on top of the three layers of active spheres used, were significantly higher. This is undoubtedly due to the fact that a two- or three-layer bed does not constitute a representative bed. Furthermore the increased ϵj values of the deep bed of spheres are the result of a stabilized turbulent condition of the air flowing through the interstices of the active spheres. In addition the presence of the inert spheres tends to minimize any channeling tendencies that could have existed in the thin beds.

NOTATION

c = concentration, lb.-moles/cu. ft.
c_p = heat capacity, B.t.u./lb. °F.
D_p = diameter of sphere, ft.
D_v = gas-film diffusivity, sq. ft./hr.
G = superficial mass velocity of flowing gas. lb./hr. sq. ft.
h_g = heat transfer coefficient for gas film, B.t.u./hr. sq. ft. °F.
k = thermal conductivity, B.t.u./hr. ft. °F.
k_g = mass transfer coefficient for gas film, based on log-mean partial pressure difference, lb.-moles/hr. sq. ft. atm.
k_x = mass transfer coefficient in binary system, lb.-moles/hr. sq. ft. atm.
j = transfer factor
j_d = mass transfer factor based on k_g, $\frac{k_g p_{gf}}{G/M}\left(\frac{\mu}{\rho D_v}\right)_f^{2/3}$
j_d^* = mass transfer factor based on Equation (14)
j_h = heat transfer factor, $\frac{h_g}{c_p G}\left(\frac{c_p \mu}{k}\right)_f^{2/3}$
M = molecular weight
N = molar flux, lb.-moles/hr. sq. ft.
N_{Pr} = Prandtl number, $c_p\mu/k$
N_{Re} = Reynolds number, $D_p G/\mu$
N_{Sc} = Schmidt number, $\mu/\rho D_v$
p = partial pressure, atm.
p_A = partial pressure of transferable component A, atm.
p_B = partial pressure of nontransferable component B, atm.
p_{gf} = partial pressure of nontransferable component in gas film, atm.
p_w = vapor pressure of water at wet-bulb temperature, atm.
q = rate of heat transfer, B.t.u./hr. sq. ft.
t = temperature, °F.
x_A = mole fraction of component A
x_B = mole fraction of component B
x_{A_o} = mole fraction of component A at the interface
x_{B_o} = mole fraction of component B at the interface

Greek Letters

Δp^* = mean partial pressure difference across gas film based on Equation (14), atm.
Δp_{ln} = log-mean partial pressure difference across gas film, atm.
$\mathcal{D}_{AB}$ = binary diffusivity for system A-B, sq. ft./hr.
ϵ = void fraction
μ = absolute viscosity, lb.$_m$/hr. ft.
π = total pressure, atm.
ρ = density, lb./cu. ft.

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